DynamicCDBM Documentation

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1 QUAKEWORX Interface

The users need to provide a single JSON parameter file to specify simulation geometry, initial damage strip, material properties, initial boundary conditions. Then QUAKEWORX will generate the mesh, run static solve and dynamic solve, user will be able to access the solution $.e$ file for postprocessing (e.g. paraview). The JSON file is shown as follows:

```
{
    "_comments": {
        "##Geometry Parameters##": {
            "xmin": "Minimum X coordinate for the domain (m)",
            "xmax": "Maximum X coordinate for the domain (m)",
            "ymin": "Minimum Y coordinate for the domain (m)",
            "ymax": "Maximum Y coordinate for the domain (m)",
            "zmin": "Minimum Z coordinate for the domain (m)",
            "zmax": "Maximum Z coordinate for the domain (m)",
            "lc": "Mesh size at far boundary (m)",
            "lc_fault": "Mesh size at near fault (m)"
        },
        "##Material Parameters##": {
            "lambda_o": "Lame Constant \lambda (Pa)",
            "shear_modulus_o": "Shear Modulus \mu (Pa)",
            "rho": "Density \rho (kg/m^3)"
        },
        "##Continuum Damage-Breakage Model Parameters##": {
            "xi_0": "Strain invariant ratio at onset of damage \xi_0",
            "xi_d": "Strain invariant ratio at onset of breakage \xi_d",
            "Cd_constant": "Damage accumulation rate C_d (1/s)",
            "CdCb_multiplier": "Breakage accumulation rate multiplier Cm (C_B (1/s) = Cm * C_d)",
            "CBH_constant": "Breakage healing rate C_BH (1/s)",
            "C_1": "Damage healing rate C_1 (1/s)",
            "C_2": "Damage healing rate C_2 (1/s)",
            "beta_width": "Width of transitional region \beta",
            "C_g": "Compliance of fluidity of the fine grain material C_g (1/(Pa s))",
            "m1": "Coefficient of power law index m_1",
            "m2": "Coefficient of power law index m_2",
            "chi": "Ratio of two energy states \chi"
        },
        "##Initial Damage Parameters##": {
            "nucl_center_x": "Nucleation center x coordinate (m)",
            "nucl_center_y": "Nucleation center y coordinate (m)",
            "nucl_center_z": "Nucleation center z coordinate (m)",
```

```
            "len_fault_strike": "Length of initial damage strip along strike direction (m)",
            "len_fault_dip": "Length of initial damage strip along dip direction (m)",
            "len_fault_normal": "Length of initial damage strip along normal direction (m)",
            "nucl_distance": "Length of nucleation patch along strike/dip direction (m)",
            "nucl_thickness": "Length of nucleation patch along normal direction (m)",
            "nucl_damage": "Value of initial damage strip",
            "e_damage": "Value of additional nucleation damage",
            "e_sigma": "Value of initial damage exponential decay",
            "duration": "Simulation time (s)"
        },
        "##Boundary Condition##": {
            "normal_traction_x": "Normal boundary traction along x direction (Pa)",
            "normal_traction_z": "Normal boundary traction along z direction (Pa)",
            "normal_traction_y": "Normal boundary traction along y direction (Pa)",
            "shear_traction": "Shear boundary traction along x-z direction (Pa)"
        },
        "##Simulation Parameters##": {
            "dt": "Simulation time step (s)",
            "end_time": "Total simulation time (s)",
            "time_step_interval": "Result saved time interval"
        }
    },
    "xmin": -10000,
    "xmax": 10000,
    "ymin": -10000,
    "ymax": 10000,
    "zmin": -10000,
    "zmax": 10000,
    "lc": 5000,
    "lc_fault": 100,
    "lambda_o": 3e10,
    "shear_modulus_o": 3e10,
    "rho": 2700,
    "xi_0": -0.8,
    "xi_d": -0.9,
    "Cd_constant": 1e4,
    "CdCb_multiplier": 1000,
    "CBH_constant": 1e4,
    "C_1": 300,
    "C_2": 0.05,
    "beta_width": 0.03,
    "C_g": 1e-10,
    "m1": 10,
    "m2": 1,
    "chi": 0.8,
    "nucl_center_x": 0,
```

```
    "nucl_center_y": -5000,
    "nucl_center_z": 0,
    "len_fault_strike": 5000,
    "len_fault_dip": 5000,
    "len_fault_normal": 1000,
    "nucl_distance": 400,
    "nucl_thickness": 200,
    "nucl_damage": 0.7,
    "e_damage": 0.3,
    "e_sigma": 250,
    "duration": 0.1,
    "normal_traction_x": 135e6,
    "normal_traction_z": 120e6,
    "normal_traction_y": 127.5e6,
    "shear_traction": 55e6,
    "dt": 1e-4,
    "end_time": 50.0,
    "time_step_interval": 1000
}
```

Most of the parameter explanations can be found in the above JSON file, for geometry setup and initial damage profile, a sketch is given as follows:

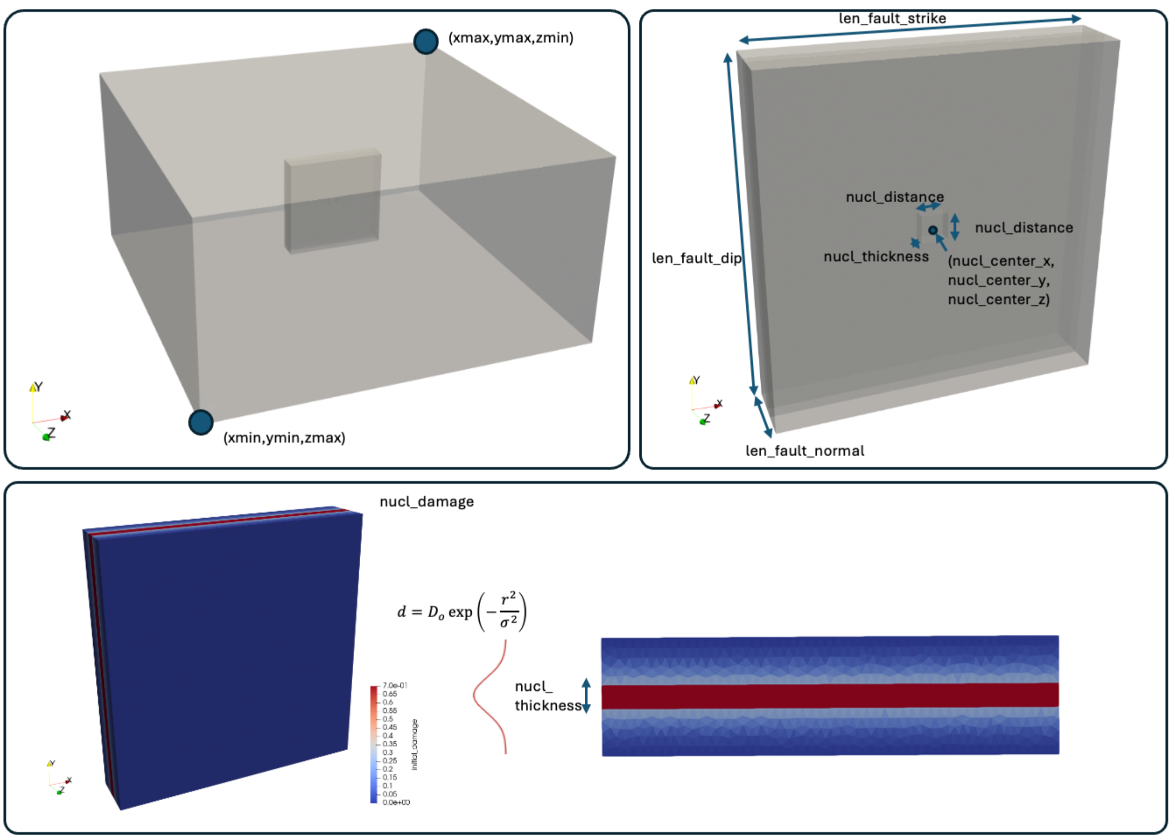

Figure 1: **Geometry and Initial Damage Profile Parameter Sketch. (1) The initial damage profile assumes constant damage value within** $nucl\_thickness$ **along normal direction, and exponentially decay along normal direction given initial damage** $D_o = nucl\_damage$ **and** e_sigma**, the additional damage** e_damage **will be applied on the patch in the center of the fault, with its length** $nucl\_distance$ **and thickness** $nucl\_thickness$**.**

2 Numerical Examples

2.1 Buried Fault

2.1.1 Geometry

The geometry, initial damage and boundary loading setup is given in Figure 2. The parameter table is given in Figure 3. The results of breakage evolution time snapshots are given in the Figure 4.

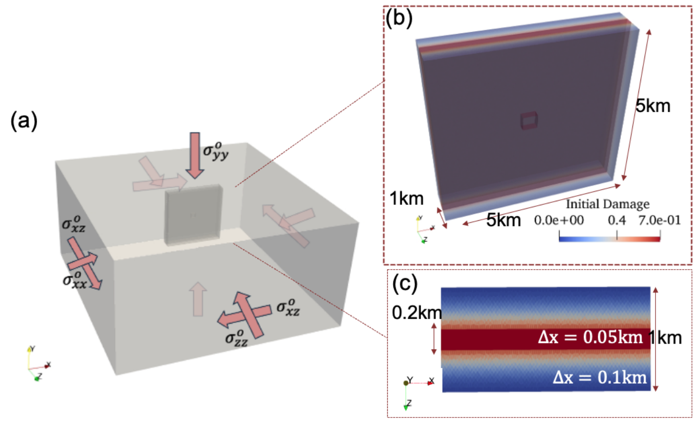

Figure 2: **Problem Setup of DynamicCDBM. A fault plane is buried in the full space. (a) Geometry setup. (b) Dimensions and initial damage distribution of fault plane. (c) Initial damage distribution of high damage zone.**

Parameter	Description
ρ	Density = 2670 kg/m^3
α	Damage Parameter (intact material 0 -- fully damaged 1)
B	Breakage Parameter (solid material 0 – granular material 1)
D	Diffusion Parameter for non-local damage $D = 0$
$\lambda(\lambda_o)$	First Lame Constant $\boldsymbol{\lambda} = \boldsymbol{\lambda_o}$ (initial value $\lambda_o = 30GPa$
$\mu(\mu_o)$	Shear Modulus $\boldsymbol{\mu} = \boldsymbol{\mu_o} + \boldsymbol{\alpha\xi_o\gamma_r}$ (initial value $\mu_o = 30GPa$)
$\gamma(\gamma_r)$	Damage Modulus $\boldsymbol{\gamma} = \boldsymbol{\alpha\gamma_r}$ (modulus at maximum damage $\alpha = 1$**)**
$\epsilon^e, \epsilon^t, \epsilon^p$	Elastic, Total, Plastic strain, respectively
I_1, I_2	First, Second elastic strain invariant $\boldsymbol{I_1} = \boldsymbol{\epsilon^e_{ij}\delta_{ij}}, \boldsymbol{I_2} = \boldsymbol{\epsilon^e_{ij}\epsilon^e_{ij}}$
ξ, ξ_o, ξ_d	Strain invariant ratio $\boldsymbol{\xi} = \boldsymbol{I_1}/\sqrt{\boldsymbol{I_2}}$, the value at onset of damage = -0.8, the value at onset of breakage = -0.9
a_0, a_1, a_2, a_3	Coefficients of polynomial construction for granular free energy $F(\xi) = a_o + a_1\xi + a_2\xi^2 + a_3\xi^3$
C_d, C_1, C_2	Rate of positive damage evolution = 10^4 1/s, Rate of damage healing: $C_1 = 300\ 1/s, C_2 = 0.05$
C_B, C_{BH}	Rate of positive breakage evolution = 10^7 1/s, Rate of breakage healing: $C_{BH} = 10^4\ 1/s$
C_g	Compliance of the fine grain granular material = 1e-10 1/(Pa s)
m_1, m_2	Coefficient of power law indexes $m_1 = 10, m_2 = 1$
χ	Energy ratio between solid and granular phase $\chi = 0.8$
σ^o_{yy}	Stress (Dip) -127.5MPa
σ^o_{xx}	Stress (Strike) -135MPa
σ^o_{xz}	Shear Stress 45MPa
σ^o_{zz}	Stress (Normal) -120MPa

Figure 3: **Parameter table for the buried fault case.**

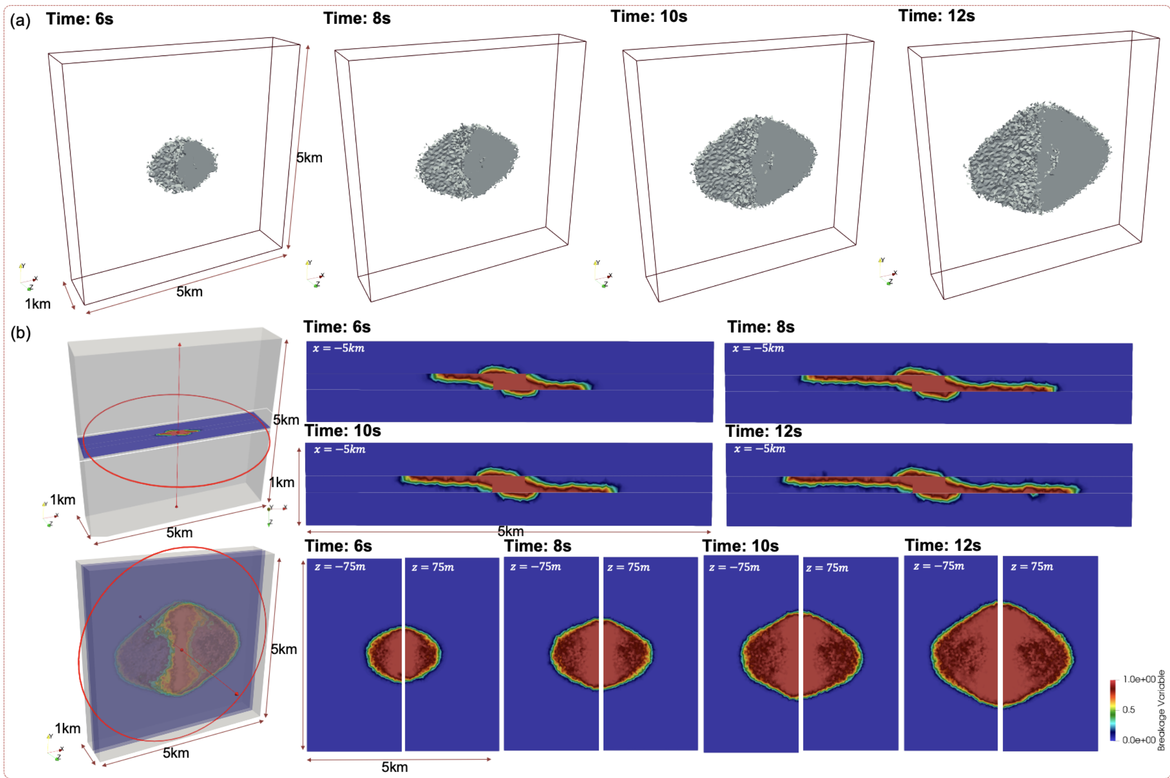

Figure 4: **Selected Time snapshots of breakage evolution for two cases in DynamicCDBM. (1) 3D Contours of breakage $B = 1$ within the damage zone. (2) Breakage evolution of XZ plane cut. The breakage within nucleation region forms its angle approximately align with maximum principal stress direction, but later its motion is restricted by the damage profile. Spontaneous propagation along with the edge of high damage region. (3) Breakage evolution of XZ planes cut, at z=-75m and z=75m. Pulse like propagation shape along depth.**

3 Theory and MOOSE Implementation

3.1 Free Energy

$$\Psi(T, \epsilon_{ij}^e, \alpha, \nabla\alpha, B) = (1 - B)\Psi_S(T, \epsilon_{ij}^e, \alpha, \nabla\alpha) + B\Psi_B(T, \epsilon_{ij}^e) \quad (1)$$

Where the total free energy Ψ is in general a function of temperature T, elastic strain ϵ^e, damage variable $\nabla\alpha$ and its spatial gradient $\nabla\alpha$, breakage variable B, and it is partitioned into elastic Ψ_S and granular Ψ_B components.

In terms of Ψ_S and Ψ_B, we consider small deformation:

$$\Psi_S = \rho(\frac{1}{2}\lambda)I_1^2 + \mu I_2 - \gamma I_1\sqrt{I_2}) \quad (2)$$

$$\Psi_B = \rho(a_o I_1 + a_1 I_1\sqrt{I_2} + a_2 I_1^2 + a_3\frac{I_1^3}{\sqrt{I_2}}) \quad (3)$$

Where the first and second strain invariant I_1 I_2 and strain invariant ratio ξ are defined as follows:

$$I_1 = tr(\epsilon^e) \quad I_2 = tr((\epsilon^e)^2) \quad \xi = \frac{I_1}{\sqrt{I_2}} \quad (4)$$

3.2 Equation for the stress

The stress S^e is derived by taking derivatives of equation (1) with respect to the elastic strain ϵ^e, the expression is given below:

$$
\begin{aligned}
S_{ij} = \frac{\partial \Psi}{\partial \epsilon^e_{ij}} &= (1-B)\frac{\partial \Psi_S}{\partial \epsilon^e_{ij}} + B\frac{\partial \Psi_B}{\partial \epsilon^e_{ij}} = (1-B)\sigma_S + B\sigma_B \\
&= (1-B)[(\lambda - \frac{\gamma}{\xi})I_1\delta_{ij} + (2\mu - \gamma\xi)\epsilon^e_{ij}] \\
&+ (B)[(2a_2 + \frac{a_1}{\xi} + 3a_3\xi)I_1\delta_{ij} + (2a_0 + a_1\xi - a_3\xi^3)\epsilon^e_{ij}]
\end{aligned} \tag{5}
$$

where the modulus are function of damage variable α:

$$\mu = \mu_o + \alpha\xi_o\gamma_r \tag{6}$$

$$\gamma = \alpha\gamma_r \tag{7}$$

Equation (5) (6) (7) is implemented in `ComputeDamageBreakageStress3D::computeQpStress()`. See `//Represent sigma (solid(s) + granular(b))`.

3.2.1 Determination of coefficients of granular phase

Please refer to (Lyakhovsky and Ben-Zion, 2014) for more details, we have four equations four unknowns:

$$
\begin{aligned}
&2a_2 + a_1/\xi_1 + 3a_3\xi_1 = 0 \\
&2a_0 + a_1\xi_1 - a_3\xi_1^3 = 0 \\
&a_0 = \chi\mu(\alpha = \alpha_{cr}|\xi = 0) \\
&a_0 + a_1\xi_d + a_2\xi_d^2 + a_3\xi_d^3 = (\mu_o + \xi_o\gamma_r) - \gamma_r\xi_d + \frac{\lambda_o}{2}\xi_d^2
\end{aligned} \tag{8}
$$

Solve the equations, we have analytical solution for a_0, a_1, a_2, a_3:

$$
\begin{aligned}
a_0 &= \chi\mu^* \\
a_1 &= \frac{-2\chi\mu^*\xi_1^3 + 6\chi\mu^*\xi_1\xi_d^2 - 4\chi\mu^*\xi_d^3 - 2\gamma_r\xi_1^3\xi_d + 2\gamma_r\xi_1^3\xi_o + \lambda_o\xi_1^3\xi_d^2 + 2\mu_o\xi_1^3}{2\xi_1^3\xi_d - 4\xi_1^2\xi_d^2 + 2\xi_1\xi_d^3} \\
a_2 &= \frac{2\chi\mu^*\xi_1^3 - 3\chi\mu^*\xi_1^2\xi_d + \chi\mu^*\xi_d^3 + 2\gamma_r\xi_1^3\xi_d - 2\gamma_r\xi_1^3\xi_o - \lambda_o\xi_1^3\xi_d^2 - 2\mu_o\xi_1^3}{\xi_1^4\xi_d - 2\xi_1^3\xi_d^2 + \xi_1^2\xi_d^3} \\
a_3 &= \frac{-2\chi\mu^*\xi_1^2 + 4\chi\mu^*\xi_1\xi_d - 2\chi\mu^*\xi_d^2 - 2\gamma_r\xi_1^2\xi_d + 2\gamma_r\xi_1^2\xi_o + \lambda_o\xi_1^2\xi_d^2 + 2\mu_o\xi_1^2}{2\xi_1^4\xi_d - 4\xi_1^3\xi_d^2 + 2\xi_1^2\xi_d^3}
\end{aligned} \tag{9}
$$

Where $\mu^* = \mu(\alpha = \alpha_{cr}|\xi = 0)$.

Equation (9) is implemented in `ComputeDamageBreakageStress3D::computecoefficients()`.

3.3 Flow rule

We express the rate of plastic part of deformation rate tensor D^p as follows:

$$\frac{\partial \epsilon^p_{ij}}{\partial t} = C_g B^{m_1}\tau_{ij}^{m_2} \tag{10}$$

Where τ_{ij} is the deviatroic stress:

$$\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{mm}\delta_{ij} \tag{11}$$

Equation (10) is implemented in `ComputeDamageBreakageStress3D::computeQpStress()`. See `/* compute strain */`

3.4 Evolution equation

The evolution equations for damage variable α and breakage variable B are given as follows:

$$\frac{\partial \alpha}{\partial t} \begin{cases} (1-B)[C_d I_2(\xi - \xi_o) + D\nabla^2 \alpha], & \xi \geq \xi_o \\ (1-B)[C_1 exp(\frac{\alpha}{C_2}) I_2(\xi - \xi_o) + D\nabla^2 \alpha], & \xi < \xi_o \end{cases} \tag{12}$$

$$\frac{\partial B}{\partial t} \begin{cases} C_B P(\alpha)(1-B) I_2(\xi - \xi_d), & \xi \geq \xi_d \\ C_{BH} I_2(\xi - \xi_d), & \xi < \xi_d \end{cases} \tag{13}$$

Equation (12) (13) is implemented in `ComputeDamageBreakageStress3D::computeQpStress()`. See `/* compute alpha and B parameters */`

3.4.1 Critical points for phase transition

3.4.2 Critical damage α_{cr}

The continuum damage-breakage model assumes the phase transition takes place when the elastic energy loss its convexity, that is one or more eigenvalues of its Hessian matrix can't maintain positive.

Table 1
Hessian matrix ($\partial^2 U / \partial \varepsilon_{ij} \partial \varepsilon_{kl}$).

	ε_{11}	ε_{22}	ε_{33}	ε_{12}	ε_{13}	ε_{23}
ε_{11}	$\lambda+2\mu-\gamma\xi+\gamma\xi e_1^2-2\gamma e_1$	$\lambda-\gamma(e_1+e_2)+\gamma\xi e_1 e_2$	$\lambda-\gamma(e_1+e_3)+\gamma\xi e_1 e_3$	0	0	0
ε_{22}	$\lambda-\gamma(e_1+e_2)+\gamma\xi e_1 e_2$	$\lambda+2\mu-\gamma\xi+\gamma\xi e_2^2-2\gamma e_2$	$\lambda-\gamma(e_2+e_3)+\gamma\xi e_2 e_3$	0	0	0
ε_{33}	$\lambda-\gamma(e_1+e_3)+\gamma\xi e_1 e_3$	$\lambda-\gamma(e_2+e_3)+\gamma\xi e_2 e_3$	$\lambda+2\mu-\gamma\xi+\gamma\xi e_3^2-2\gamma e_3$	0	0	0
ε_{12}	0	0	0	$2\mu-\gamma\xi$	0	0
ε_{13}	0	0	0	0	$2\mu-\gamma\xi$	0
ε_{23}	0	0	0	0	0	$2\mu-\gamma\xi$

Here $e_i = \varepsilon_i / \sqrt{I_2}$ is a normalized value of the deformation along the principal axis "i".

Figure 5: **Hessian matrix for solid phase, figure from (Lyakhovsky et al., 2011)**

The conditions for positive eigenvalues are (also obtained from (Lyakhovsky et al., 2011):

$$\begin{aligned} &(2\mu - \gamma\xi)^2 + (2\mu - \gamma\xi)(3\lambda - \gamma\xi) + (\lambda\gamma\xi - \gamma^2)(3 - \xi^2) > 0 \\ &(2\mu - \gamma\xi) > 0 \end{aligned} \tag{14}$$

If we set the first condition equals to zero, it corresponds to [15a], setting the second condition equals to zero is related to [15b] in Figure 6. From equation (14), we obtain the analytical solution for critical damage variable α_{cr} as follows:

$$\begin{cases} \alpha_{cr} = 1, \quad \xi < \xi_o \\ \alpha_{cr} = \frac{\lambda\xi^3 - 6\lambda\xi_o + 6\mu_o\xi - 8\mu_o\xi_o \pm \sqrt{\lambda^2\xi^6 - 12\lambda^2\xi^3\xi_o + 36\lambda^2\xi_o^2 + 12\lambda\mu_o\xi^4 - 16\lambda\mu_o\xi^3\xi_o - 72\lambda\mu_o\xi^2 + 72\lambda\mu_o\xi\xi_o + 72\lambda\mu_o - 12\mu_o^2\xi^2 + 48\mu_o^2}}{2\gamma_r(3\xi^2 - 6\xi\xi_o + 4\xi_o^2 - 3)} \\ \quad \xi > \xi_o \;\; and \;\; \xi < \xi_1 \\ \alpha_{cr} = \frac{2\mu_o}{\gamma_r(\xi - 2\xi_o)} \quad \xi > \xi_1 \end{cases} \tag{15}$$

Equation (12) (13) is implemented in `ComputeDamageBreakageStress3D::alphacr_root1()` and `alphacr_root2()`.

3.4.3 Critical strain invariant ratio ξ_1

As shown in Figure 6, point b is a critical point where three phases (solid, granular, pseudo-gas) coexist. The conditions for eigenvalues of the Hessian matrix vanishes lead to the simplifications of the conditions given in equation 14:

$$\begin{aligned} \lambda - \frac{\gamma}{\xi} = 0 \\ 2\mu - \gamma\xi = 0 \end{aligned} \tag{16}$$

Using the equations of shear modulus and damage modulus given above (equations (6) (7)), we obtain analytical solution for ξ_1, which is the strain invariant ratio at this critical point where three phases coexist:

$$\xi_1 = \xi_o + \sqrt{\xi_o^2 + 2\frac{\mu_o}{\lambda_o}} \tag{17}$$

Equation (12) (13) is implemented in `ComputeDamageBreakageStress3D::computeQpStress()`. See `//compute xi_1`.

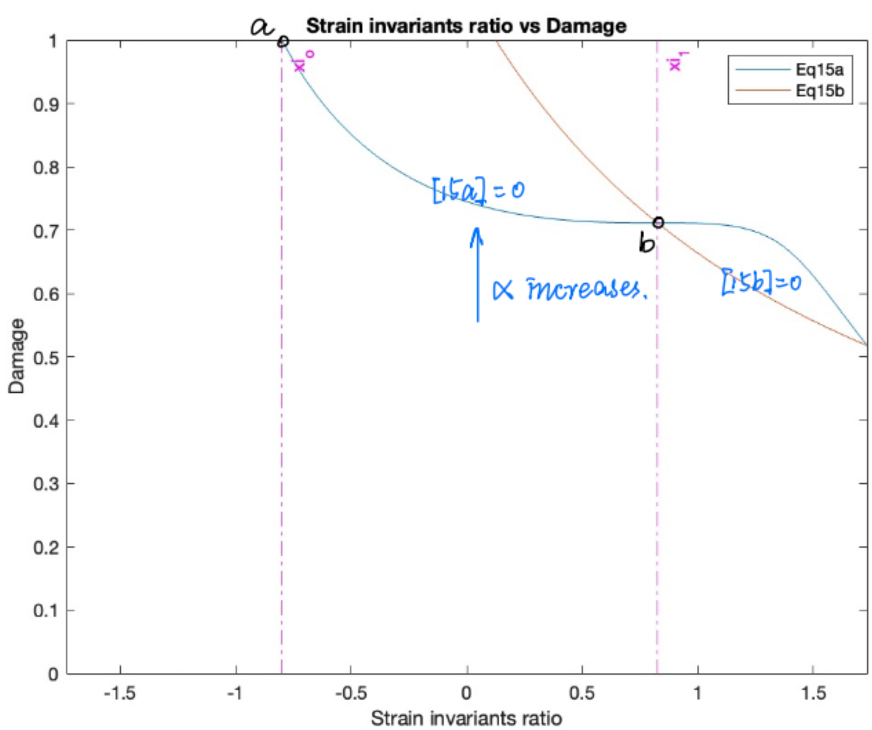

Figure 6: **Phase diagram: damage variable α vs strain invariant ratio ξ**

3.4.4 Critical damage modulus γ_r

The critical (or maximum) damage modulus γ_r represents the damage modulus at maximum damage variable $\alpha = 1$ and onset of damage accumulation strain state $\xi = \xi_0$. From first equation (14) and Figure 6 point a, we plug in $\alpha = 1$ and $\xi = \xi_0$ into the first equation and solve for γ_r:

$$\gamma_r = \frac{-\xi_o\left(-\lambda\xi_o^2 + 6\lambda + 2\mu_0\right) - \sqrt{\left(\lambda\xi_o^2 + 2\mu_0\right)\left(\lambda\xi_o^4 - 12\lambda\xi_o^2 + 36\lambda - 6\mu_0\xi_o^2 + 24\mu_0\right)}}{2\left(\xi_o^2 - 3\right)} \tag{18}$$

Equation (12) (13) is implemented in `ComputeDamageBreakageStress3D::computeQpStress()`. See `computegammar()`.

References

Lyakhovsky, V., Ben-Zion, Y., 2014. A continuum damage–breakage faulting model and solid-granular transitions. Pure and Applied Geophysics 171, 3099–3123.

Lyakhovsky, V., Hamiel, Y., Ben-Zion, Y., 2011. A non-local visco-elastic damage model and dynamic fracturing. Journal of the Mechanics and Physics of Solids 59, 1752–1776.